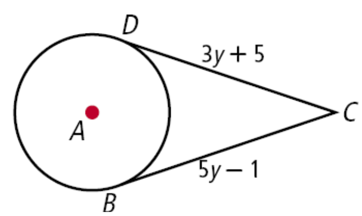
Attendance Problems. Solve for x.

1. $\frac{x}{5} = \frac{28}{35}$

2. $3x = 12^2$

3. $\overline{BC}$ and $\overline{DC}$ are tangent to $\bigcirc A$. Find BC.

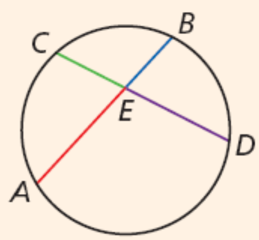


- I can find the lengths of segments formed by lines that intersect circles.
- I can use the lengths of segments in circles to solve problems.

Vocabulary		
secant segment	external secant segment	tangent segment

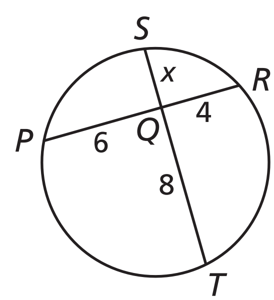
<u>Common Core</u>: CC.9-12.G.C.2 Identify and describe relationships among inscribed angles,radii, and chords.

Theorem 11-6-1 Chord-Chord Product Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.	 Chords $\overline{AB}$ and $\overline{CD}$ intersect at E.	$AE \cdot EB = CE \cdot ED$

1 Applying the Chord-Chord Product Theorem

Find the value of x and the length of each chord.

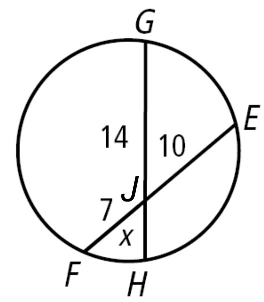


$$PQ \cdot QR = SQ \cdot QT$$
$$6(4) = x(8)$$
$$24 = 8x$$
$$3 = x$$
$$PR = 6 + 4 = 10$$
$$ST = 3 + 8 = 11$$

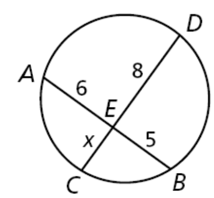
Question: How many mathematicians to does it take to tie up a package?
Answer: None, you just need a chord.

"The poor man is not he who is without a cent, but he who is without a dream."—*Poet, Harry Kemp*

Example 1. Find the value of *x* and the length of each chord.

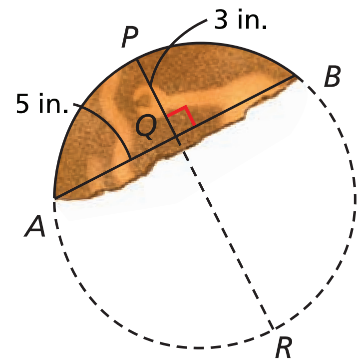


4. **Guided Practice**. Find the value of x and the length of each chord.



2 Archaeology Application

Archaeologists discovered a fragment of an ancient disk. To calculate its original diameter, they drew a chord $\overline{AB}$ and its perpendicular bisector $\overline{PQ}$. Find the disk's diameter.



Since $\overline{PQ}$ is the perpendicular bisector of a chord, $\overline{PR}$ is a diameter of the disk.

$$AQ \cdot QB = PQ \cdot QR$$

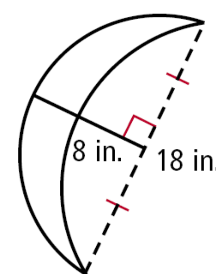
$$5(5) = 3(QR)$$

$$25 = 3QR$$

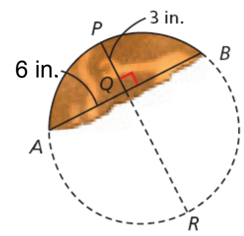
$$8\frac{1}{3}\text{ in.} = QR$$

$$PR = 3 + 8\frac{1}{3} = 11\frac{1}{3}\text{ in.}$$

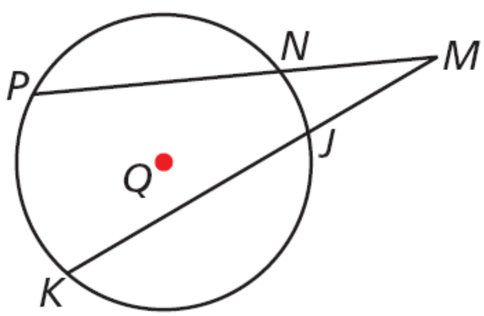
Example 2. The art department is contracted to construct a wooden moon for a play. One of the artists creates a sketch of what it needs to look like by drawing a chord and its perpendicular bisector. Find the diameter of the circle used to draw the outer edge of the moon.



5. <u>Guided Practice</u>. Suppose the length of chord AB that the archeologists drew was 12 in. In this case how much longer is the disk's diameter compared to the disk on p. 793?

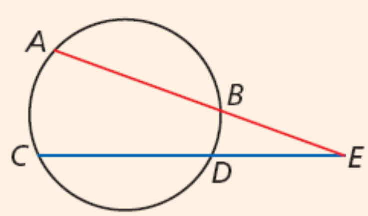


A **<u>secant segment</u>** is a segment of a secant with at least one endpoint on the circle. An **<u>external secant segment</u>** is a secant segment that lies in the exterior of the circle with one endpoint on the circle.



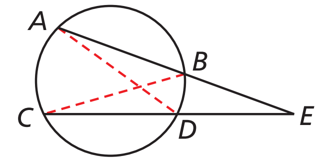
$\overline{PM}$, $\overline{NM}$, $\overline{KM}$, and $\overline{JM}$ are secant segments of $\odot Q$. $\overline{NM}$ and $\overline{JM}$ are external secant segments.

Theorem 11-6-2 **Secant-Secant Product Theorem**

THEOREM	HYPOTHESIS	CONCLUSION
If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. (whole • outside = whole • outside)	 Secants $\overline{AE}$ and $\overline{CE}$ intersect at E.	$AE \cdot BE = CE \cdot DE$

PROOF

Secant-Secant Product Theorem



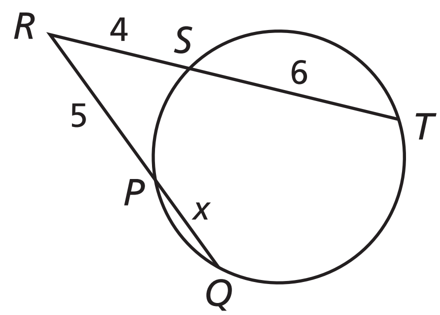
Given: Secant segments $\overline{AE}$ and $\overline{CE}$

Prove: $AE \cdot BE = CE \cdot DE$

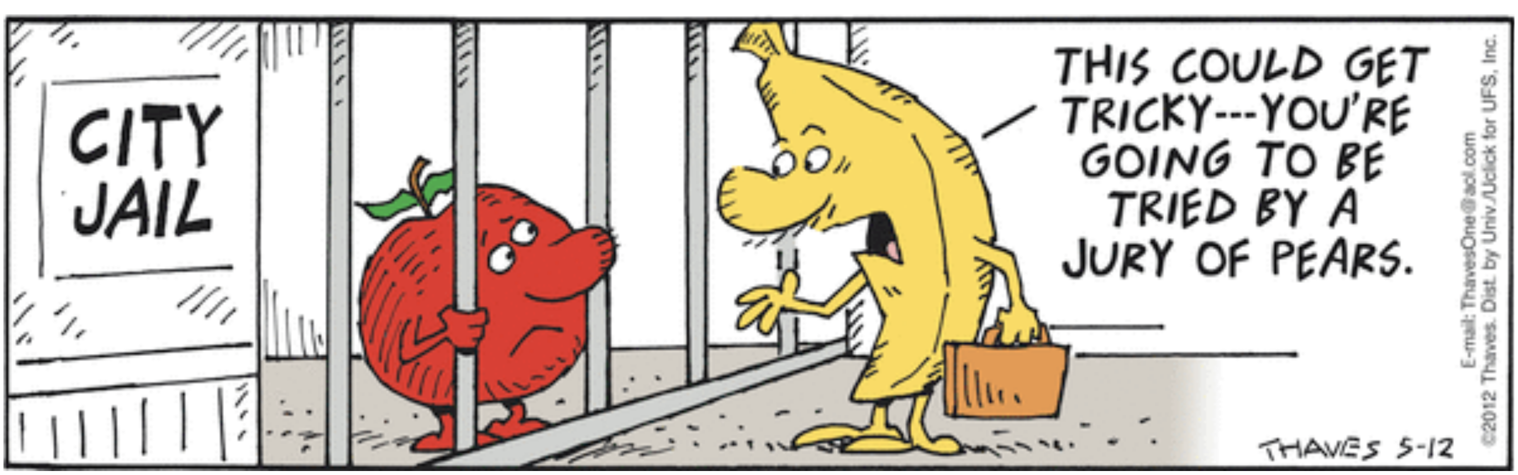
Proof: Draw auxiliary line segments $\overline{AD}$ and $\overline{CB}$. $\angle EAD$ and $\angle ECB$ both intercept $\overset{\frown}{BD}$, so $\angle EAD \cong \angle ECB$. $\angle E \cong \angle E$ by the Reflexive Property of $\cong$. Thus $\triangle EAD \sim \triangle ECB$ by AA Similarity. Therefore corresponding sides are proportional, and $\frac{AE}{CE} = \frac{DE}{BE}$. By the Cross Products Property, $AE \cdot BE = CE \cdot DE$.

3 Applying the Secant-Secant Product Theorem

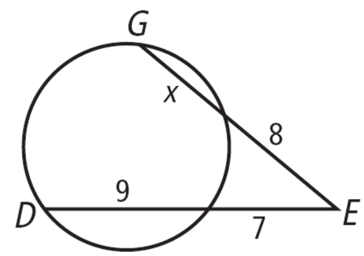
Find the value of x and the length of each secant segment.



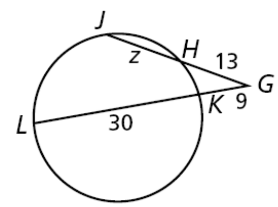
$$RT \cdot RS = RQ \cdot RP$$
$$10(4) = (x + 5)5$$
$$40 = 5x + 25$$
$$15 = 5x$$
$$3 = x$$
$$RT = 4 + 6 = 10$$
$$RQ = 5 + 3 = 8$$



Example 3. Find the value of *x* and the length of each secant segment.

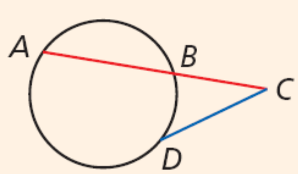


6. <u>Guided Practice</u>. Find the value of z and the length of each secant segment.

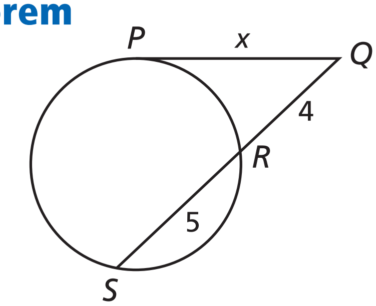


12-6 Segment Relationships in Circles: *(p 844)* 13-15, 17, 18.

Theorem 11-6-3 **Secant-Tangent Product Theorem**

THEOREM	HYPOTHESIS	CONCLUSION
If a secant and a tangent intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. (whole • outside = tangent²)	 Secant $\overline{AC}$ and tangent $\overline{DC}$ intersect at C.	$AC \cdot BC = DC^2$

4 Applying the Secant-Tangent Product Theorem

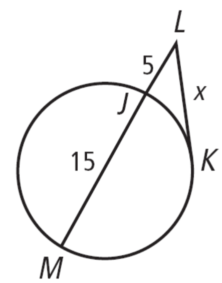


Find the value of x.

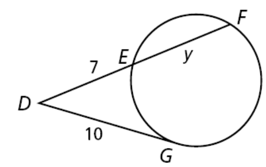
$$SQ \cdot RQ = PQ^2$$
$$9(4) = x^2$$
$$36 = x^2$$
$$\pm 6 = x$$

The value of x must be 6 since it represents a length.

Example 4. Find the value of x.



7. Find the value of y.



12-6 Segment Relationships in Circles: *(p 844)* 13-15, 17-19, 21, 22, 24, 27, 30-35.